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Minimum-Rank Optimal Update of Elemental Stiffness Parameters for Structural Damage Identification

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A new optimal update method for the correlation of dynamic structural finite element models with modal data is presented. The method computes a minimum-rank solution for the perturbations of the elemental stiffness parameters while constraining the connectivity of the global stiffness matrix. The resulting model contains a more accurate representation of the dynamics of the test structure, and the changes between the original model and the updated model can be interpreted as modeling errors or as changes in the structure resulting from damage. The motivation for the method is presented in the context of existing optimal matrix update procedures. This method is distinct from past minimum-rank optimal update procedures because it computes minimum-rank updates directly to the elemental stiffness parameters. The method is demonstrated numerically on a spring-mass system and is also applied to experimental data from the NASA Langley Research Center eight-bay truss damage detection experiment. The results demonstrate that the proposed procedure may be useful for updating elemental stiffness parameters in the context of damage detection and model refinement.

Nomenclature

$[A]$	= stiffness connectivity matrix
E, I	= Young's modulus, cross-sectional moment of inertia
$[M]$, $[D]$, $[K]$	= structural mass, damping, and stiffness matrices of correct model
$[M_u]$, $[D_u]$, $[K_u]$	= structural mass, damping, and stiffness matrices of nominal model
n_d	= number of degrees of freedom
n_p	= number of elemental stiffness parameters
$\{p\}$, $[P]$	= vector and diagonal matrix of element-level stiffness parameters
$[\Delta M]$, $[\Delta D]$, $[\Delta K]$	= structural mass, damping, and stiffness matrix perturbations
ω_j, $\{\phi_j\}$, $\{E_j\}$	= circular modal frequency, mode shape, and force error for jth mode

Introduction

MODIFICATION of a structural finite element model (FEM) such that the FEM eigensolution matches the results of a modal vibration experiment is a subject that has received much attention in the literature in recent years. Methods for this type of FEM updating are applicable to problems such as model refinement, for better prediction of structural static and dynamic response, for structural damage detection, and for location of cracks and failures in structures such as aircraft skin, bridge supports, and offshore oil platforms.

One class of methods for correlating measured modal data with analytical finite element models is the minimization or elimination of modal force error, which is the error resulting from the substitution of the analytical FEM and the measured modal data into the structural eigenproblem. Various methods have been developed to minimize or to eliminate some measure of the error in the eigenproblem by perturbing the baseline values in the analytical model, such as the components of the stiffness, damping, and mass matrices. One type of method, known as sensitivity-based model update, uses the sensitivities of the modal response parameters of the FEM (such as modal frequencies and mode shapes) to the structural design parameters (such as Young's modulus, density, etc.) to iteratively minimize the modal force error.[1–3] Another type of method, known as eigenstructure assignment, designs a controller that minimizes the modal force error. The controller gains are then interpreted in terms of structural parameter modifications.[4] Still another type of method, known as optimal matrix update, solves a closed-form equation for the matrix perturbations that minimize the modal force error or constrain the solution to satisfy it.[5–12] It is this type of method that is of interest in this paper. Similar methods to implement optimal matrix update using frequency response functions have been developed[13] but will not be included in the current discussion due to the difference in formulation compared with the modal-based update procedures. Much of the research done in optimal matrix update has focused on estimating perturbations to the mass, damping, and stiffness properties. In the context of structural damage detection and health monitoring, the perturbations to the stiffness properties are usually the most relevant. In this paper, only the perturbation of the structural stiffness properties will be considered.

Computing the stiffness property perturbations that eliminate the modal force error is often an underdetermined problem, since the number of unknowns in the perturbation set can be much larger than the number of measured modes and the number of measurement degrees of freedom (DOF). In this case, the property perturbations that satisfy the modal force error equation are nonunique. Optimal matrix update methods thus apply a minimization to the property perturbation to select a solution to the modal force error equation subject to constraints such as symmetry, positive definiteness, and sparsity. Typically, this minimization applies to either a norm or the rank of the perturbation property matrix or vector.

The main distinction between optimal update methods that minimize some measure of the stiffness property perturbations can be drawn based on two characteristics: 1) the stiffness property that is varied and 2) the objective function that is used to select the solution. The stiffness properties can be categorized as the global stiffness matrix, the elemental stiffness matrices, or the elemental stiffness parameters (e.g., E, I, etc.). The objective functions are either the minimum of a norm of the property perturbation or the minimum of the rank of the property perturbation. Table 1 shows how several of the most widely known optimal matrix update procedures can be categorized according to these characteristics. The columns in this

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Table 1 Characteristics of optimal matrix update methods

	Updated properties		
Criteria	Global matrix	Elemental matrix	Elemental parameter
Min. norm	Ref. 5 Ref. 6 Ref. 7 Ref. 8	Ref. 10	Ref. 14 Ref. 15
Min. rank	Ref. 11	Ref. 12	This paper

table categorize methods (and cite examples from the literature) according to which model parameter is used in the update procedure. The rows categorize the methods by whether a minimum-norm (e.g., least-squares) or a minimum-rank function is used as the objective of the optimization.

As shown in Table 1, the majority of the early work in optimal matrix update used the minimum-norm perturbation of the global stiffness matrix.[5–8] The motivation for using this objective function is that the desired perturbation is the one that is smallest in overall magnitude. Later work by Kaouk and Zimmerman,[11] as shown in the second row of Table 1, used the minimum-rank perturbation of the global stiffness matrix. This was motivated by the application of damage detection, where the perturbations could be assumed to be limited to a few isolated locations. The minimum-rank stiffness matrix perturbation can be thought of as the stiffness matrix perturbation with the smallest number of nonzero values. An extension of this work computes the perturbations at the element stiffness matrix level, to limit the computed perturbations to certain structural DOF.[12]

A common drawback of the methods listed in the first two columns of Table 1 is that the computed perturbations are made to stiffness matrix values at the structural DOF rather than at the element stiffness parameter level. There are three main advantages to computing perturbations to the elemental stiffness parameters rather than to stiffness matrix entries. 1) The resulting updates have direct physical relevance and thus can be more easily interpreted in terms of structural damage or errors in the FEM. 2) The connectivity of the FEM is preserved, so that the resulting updated FEM has the same load path set as the original. 3) A single parameter that affects a large number of structural elements can be varied independently. This advantage is especially relevant, for example, in civil engineering applications, where a parameter such as the Young's modulus of concrete may be uniform throughout a number of elements but not precisely known. Previous techniques to compute perturbations at the element parameter level have been proposed by Chen and Garba[14] and Li and Smith.[15] These techniques use the sensitivity of the entries in the stiffness matrix to the elemental stiffness parameters so that the minimum norm criterion can be applied directly to the vector of elemental stiffness parameters. Thus, the resulting update consists of a vector of elemental stiffness parameters that is a minimum norm solution to the optimal update equation. The stiffness parameters that are updated can be limited in the construction of the sensitivity matrix.

The method proposed in this paper, also listed in Table 1, computes perturbations to the element-level stiffness parameters by solving the optimal update equation for the stiffness matrix subject to a minimum-rank objective function. For convenience, the new technique is termed the minimum-rank elemental update (MREU). This method is designed to exploit the advantages of both the minimum-rank solution technique and the computation of element-level stiffness perturbations. The remainder of the paper is organized as follows. First, the theory of optimal matrix update is summarized, including brief outlines of the existing methods. Second, the theoretical development of the MREU is presented, followed by a method for computing the required "stiffness connectivity matrix." Next, the method is demonstrated using a numerical example, followed by application to experimental data from a NASA truss damage-detection experiment. The results of the MREU are compared to the results of a minimum-norm elemental stiffness update using this experimental data set.

Theory of Optimal Matrix Update

The basic theory of optimal matrix update techniques begins with the second-order structural equation of motion

$$[M]\{\ddot{x}\} + [D]\{\dot{x}\} + [K]\{x\} = \{F\} \tag{1}$$

The eigensolution of this equation with no externally applied forces represents the free vibration of the structure. For the jth structural vibration mode, this is expressed as

$$\left(-\omega_j^2[M] + i\omega_j[D] + [K]\right)\{\phi_j\} = \{0\} \tag{2}$$

Now presuming that the structural model matrices contain some error, perhaps because of modeling errors or changes in the structure such as damage, the model matrices can be related to the true matrices as

$$\begin{aligned} [M] &= [M_u] - [\Delta M] \\ [D] &= [D_u] - [\Delta D] \\ [K] &= [K_u] - [\Delta K] \end{aligned} \tag{3}$$

where the matrices with subscript u are the nominal model matrices and the Δ matrices are the matrix perturbations. Substituting Eq. (3) into Eq. (2) and moving the perturbation terms to the right-hand side yield

$$\begin{aligned} &-\left(\omega_j^2[M_u] + i\omega_j[D_u] + [K_u]\right)\{\phi_j\} \\ &\quad = \left(-\omega_j^2[\Delta M] + i\omega_j[\Delta D] + [\Delta K]\right)\{\phi_j\} \end{aligned} \tag{4}$$

Since all of the terms on the left-hand side of Eq. (4) are known, the modal force error $\{E_j\}$ can be defined for each measured mode as

$$\{E_j\} = \left(-\omega_j^2[M_u] + i\omega_j[D_u] + [K_u]\right)\{\phi_j\} \tag{5}$$

After determining $\{E_j\}$, the matrix perturbations can be computed by solving

$$\left(-\omega_j^2[\Delta M] + i\omega_j[\Delta D] + [\Delta K]\right)\{\phi_j\} = \{E_j\} \tag{6}$$

for $[\Delta M]$, $[\Delta D]$, and $[\Delta K]$. Under the assumptions $[\Delta M]$ and $[\Delta D] = 0$, Eq. (6) simplifies to

$$[\Delta K]\{\phi_j\} = \{E_j\} \tag{7}$$

As described in the Introduction, examples of the methods used to solve Eq. (7) are categorized in Table 1. A brief summary of the mathematical formulations of these methods follows.

The minimum-norm perturbation of the global stiffness matrix was the approach used in Refs. 5–8. As described by Smith and Beattie,[8] this approach can be summarized as

$$\min \|\Delta K\| \tag{8}$$

subject to the constraints of Eq. (7) and $[\Delta K]$ symmetric and sparse. Constraining the sparsity to be the same as the nominal FEM stiffness matrix has the effect of ensuring that no new load paths are generated by the updated model.

The minimum-rank perturbation approach of Kaouk and Zimmerman[11] can be summarized as

$$\min\{\text{rank}([\Delta K])\} \tag{9}$$

subject to the constraints of Eq. (7) and $[\Delta K]$ symmetric and positive definite. An extension of this method partitions the perturbation matrix so that only the elemental stiffness matrices associated with certain structural DOF are updated.[12]

The minimum-norm, element-level update procedures presented by Chen and Garba[14] and Li and Smith[15] incorporate the connectivity constraint between the element-level stiffness parameters and the entries in the global stiffness matrix directly into Eq. (7). This can be derived by writing $[\Delta K]$ as the first-order term in a Taylor series expansion of $[K]$ about $\{p\}$, the vector of element-level stiffness parameters,

$$[\Delta K] = \frac{\partial}{\partial p}[K]\{\Delta p\} \tag{10}$$

and then substituting Eq. (10) into Eq. (7) and rearranging to get

$$\frac{\partial}{\partial p}([K]\{\phi_j\})\{\Delta p\} = \{E_j\} \tag{11}$$

The minimum-norm $\{\Delta p\}$ (in the underdetermined case) or the $\{\Delta p\}$ that produces the least-squares error (in the overdetermined case) are then solved for from Eq. (11).

The MREU technique uses an approach similar to that presented by Chen and Garba[14] and Li and Smith,[15] by including the connectivity constraint directly into the modal force error equation, but uses a minimum-rank solution, as presented by Kaouk and Zimmerman,[11] to solve for the elemental parameters. The derivation of the MREU is presented in the following section.

Derivation of Minimum-Rank Elemental Parameter Update

The derivation of the MREU technique begins with the parameterization of the $(n_d \times n_d)$ global stiffness matrix $[K]$ as

$$[K] = [A][P][A]^T \tag{12}$$

where the $(n_d \times n_p)$ matrix $[A]$ is defined as the stiffness connectivity matrix and the $(n_p \times n_p)$ diagonal matrix $[P]$ has the elemental stiffness parameters of the $(n_p \times 1)$ vector $\{p\}$ as its diagonal entries. Mathematically, this is defined as

$$\mathrm{diag}([P]) = \{p\} \tag{13}$$

The formulation of Eq. (12) arises from the generalized form of the elemental stiffness matrix, presented by Zienkiewicz[16] as

$$[K^e] = \int_{V^e} [B]^T[D][B]\,\mathrm{d}V \tag{14}$$

where $[D]$ is a function purely of the material properties, $[B]$ is a function of the elemental strain-displacement relations, and V^e is the elemental volume. Because the global stiffness matrix is an algebraic assembly of the elemental stiffness matrices, it can also be written in the form of Eq. (14). For linear isotropic elements, Eq. (14) can be rewritten in the form of Eq. (12). Therefore, because the global stiffness matrix is a linear function of the elemental stiffness parameters, and assuming $[A]$ is independent of $[P]$ (which is discussed later), Eq. (12) can be perturbed to get

$$[K + \Delta K] = [A][P + \Delta P][A]^T \tag{15}$$

Expanding Eq. (15) and subtracting Eq. (12) from it yields the parameterization of the perturbed global stiffness matrix $[\Delta K]$,

$$[\Delta K] = [A][\Delta P][A]^T \tag{16}$$

The connectivity constraint is enforced by substituting Eq. (16) into Eq. (7) to get

$$[A][\Delta P][A]^T\{\phi_j\} = \{E_j\} \tag{17}$$

To put Eq. (17) in the proper form for minimum-rank solution, first perform a minimum-norm solution of Eq. (17) for $[\Delta P][A]^T\{\phi_j\}$ to get

$$[\Delta P][A]^T\{\phi_j\} = \{Q_j\} \tag{18}$$

The solution of Eq. (18) for symmetric minimal rank $[\Delta P]$ is given in Ref. 11 as

$$[\Delta P] = \{Q_j\}h\{Q_j\}^T \tag{19}$$

where

$$h = \left(\{Q_j\}^T[A]^T\{\phi_j\}\right)^{-1} \tag{20}$$

In the case when multiple modes are used in the update, h becomes the matrix $[H]$, the modal vectors $\{\phi_j\}$ are collected into the columns of the matrix $[\Phi]$, and the vectors $\{Q_j\}$ are collected into the columns of the matrix $[Q]$. The equivalent forms of Eqs. (19) and (20) become

$$[\Delta P] = [Q][H][Q]^T \tag{21}$$

and

$$[H] = ([Q]^T[A]^T[\Phi])^{-1} \tag{22}$$

Note that because $[A]$ is typically very sparse (95–98%), it is most efficient to use the algorithms from a sparse linear algebra library [such as those contained in MATLAB™ (Ref. 17)] to compute $\{Q\}$.

In the special case where the entire FEM global DOF set is used in the analysis, $[A]$ is independent of the elemental stiffness parameters $[P]$. However, under most practical circumstances, the number of measured DOF is much smaller than the number of FEM DOF. One approach used to overcome this limitation is to define a reduced DOF set for the FEM, such as that obtained using Guyan condensation.[18] In this case, the connectivity matrix $[A]$ becomes a function of the stiffness parameters $[P]$, and so $[\Delta P]$ must be computed iteratively. This can be seen by partitioning Eq. (12) into master DOF, represented by m, and omitted DOF, represented by o, to obtain

$$[K] = \begin{bmatrix} K_{mm} & K_{mo} \\ K_{mo}^T & K_{oo} \end{bmatrix} = \begin{bmatrix} A_m \\ A_o \end{bmatrix}[P]\begin{bmatrix} A_m \\ A_o \end{bmatrix}^T \tag{23}$$

which can be simplified to

$$\begin{bmatrix} K_{mm} & K_{mo} \\ K_{mo}^T & K_{oo} \end{bmatrix} = \begin{bmatrix} A_m P A_m^T & A_m P A_o^T \\ A_o P A_m^T & A_o P A_o^T \end{bmatrix} \tag{24}$$

Then the Guyan-reduced stiffness matrix can be written as

$$\begin{aligned} \bar{K} &= \begin{bmatrix} I \\ -K_{oo}^{-1}K_{mo}^T \end{bmatrix}^T [K] \begin{bmatrix} I \\ -K_{oo}^{-1}K_{mo}^T \end{bmatrix} \\ &= \begin{bmatrix} I \\ -\left(A_o P A_o^T\right)^{-1}\left(A_o P A_m^T\right) \end{bmatrix}^T [A][P][A]^T \\ &\times \begin{bmatrix} I \\ -\left(A_o P A_o^T\right)^{-1}\left(A_o P A_m^T\right) \end{bmatrix} = [\bar{A}][P][\bar{A}]^T \end{aligned} \tag{25}$$

where the condensed connectivity matrix $[\bar{A}]$ is a function of both the full connectivity matrix $[A]$ and the stiffness parameters contained in $[P]$. It is evident that the linear perturbation required for Eqs. (15) and (16) will no longer be valid for the reduced system. Thus, Guyan reduction destroys the simple linear equation solution.

An alternative to reducing the number of FEM DOF is to expand the measured mode shapes to the FEM DOF. This can be done using a linear projection onto the eigensolution of the baseline model, as implemented by Farhat and Hemez.[2] The consequences of the test/analysis DOF mismatch are not further discussed in this paper.

Computation of Stiffness Connectivity Matrix

The stiffness connectivity matrix, as defined in Eq. (12), provides a transformation from the elemental stiffness parameters to the global system DOF. It can be computed from a sensitivity analysis of the global stiffness matrix using the following procedure. Recognizing that $[\Delta P]$ is diagonal, one can rewrite Eq. (16) in tensor form (as shown in Ref. 19), so that the (i, j) entry in the global stiffness matrix can be parameterized as

$$\Delta K_{ij} = \lfloor A_{i1}A_{j1} \quad A_{i2}A_{j2} \quad \ldots \rfloor \begin{Bmatrix} \Delta p_1 \\ \Delta p_2 \\ \ldots \end{Bmatrix} \tag{26}$$

This relationship can also be expressed using the sensitivity of the (i, j) entry in the global stiffness matrix to the βth elemental stiffness parameter, i.e.,

$$\Delta K_{ij} = \frac{\partial K_{ij}}{\partial p_\beta}(\Delta p_\beta) \tag{27}$$

Comparing Eq. (27) with Eq. (26) yields an equivalence between the entries of the sensitivity matrix and the stiffness connectivity matrix, which can be written

$$A_{i\beta}A_{j\beta} = \frac{\partial K_{ij}}{\partial p_\beta} \tag{28}$$

If the finite elements used in the model are linear functions of the elemental stiffness parameters, this sensitivity can be computed using a finite difference approach as

$$\frac{\partial K_{ij}}{\partial p_\beta} = \frac{\Delta K_{ij}}{\Delta p_\beta} \tag{29}$$

The stiffness connectivity matrix $[A]$ can then be obtained by computing the sensitivity matrix using Eq. (29) and then solving Eq. (28) for each entry in the connectivity matrix.

Note that the stiffness connectivity matrix $[A]$ is equivalent to that defined by Peterson et al.[19] In that paper, $[A]$ is computed algebraically using element-level expressions for the connectivity. The matrix obtained using the aforementioned sensitivity computations should be equivalent to the matrix obtained using the algebraic method. The sensitivity-based method for computing the connectivity matrix was developed as an alternative to the algebraic method, because the algebraic method basically requires the development of a finite element assembly code to assemble the element-level connectivity matrices into the global connectivity matrix. The sensitivity-based method uses the features of a commercial finite element code to complete the assembly. The algebraic method is more theoretically elegant, but for large problems with complex geometry, the sensitivity method is more easily implemented.

Numerical Example

To demonstrate the numerical implementation of the MREU procedure, it is applied to the spring-mass system shown in Fig. 1. This example demonstrates that an unknown set of stiffness parameter perturbations can be computed using only the mass and stiffness matrix of the nominal model and the measured modal frequencies and mode shapes of the perturbed system. Consider the nominal model of the system to have the parameters

$$\{k_1, k_2, k_3, k_4\} = \{1, 1, 1, 1\} \qquad \{m_1, m_2, m_3\} = \{1, 1, 1\} \tag{30}$$

which has the mass and stiffness matrices

$$[K_u] = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \qquad [M_u] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{31}$$

A sensitivity analysis of this stiffness matrix $[K_u]$ to the four stiffness parameters $\{k_1, k_2, k_3, k_4\}$ yields a connectivity matrix $[A]$ of

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \tag{32}$$

Now consider a perturbed set of stiffness parameters and perturbed stiffness matrix

$$\{k_1, k_2, k_3, k_4\} = \{1, 0.8, 1, 1\}$$

$$[K] = \begin{bmatrix} 1.8 & -0.8 & 0 \\ -0.8 & 1.8 & -1 \\ 0 & -1 & 2 \end{bmatrix} \tag{33}$$

Computing the modal parameters of the perturbed system using the perturbed stiffness matrix from Eq. (33) and the nominal mass matrix from Eq. (31) yields

$$\{\omega_1, \omega_2, \omega_3\} = \{0.7586, 1.3703, 1.7740\}$$

$$[\{\phi_1\}, \{\phi_2\}, \{\phi_3\}] = \begin{bmatrix} 0.4715 & -0.7815 & 0.4086 \\ 0.7218 & 0.0758 & -0.6880 \\ 0.5067 & 0.6193 & 0.5998 \end{bmatrix} \tag{34}$$

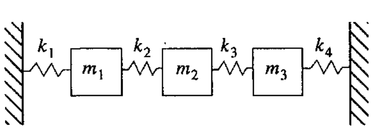

Fig. 1 Spring-mass system for numerical example.

Assume that only the first mode is measured, so that ω_1 and $\{\phi_1\}$ from Eq. (34) are known. The modal force error $\{E_1\}$ for this mode is computed by substituting $[K_u]$, $[M_u]$, ω_1, and $\{\phi_1\}$ into Eq. (5) to get

$$\{E_1\} = \begin{Bmatrix} -0.0501 \\ 0.0501 \\ 0 \end{Bmatrix} \tag{35}$$

The MREU equation can then be formed as in Eq. (18) to produce

$$[\Delta p] \begin{Bmatrix} 0.4715 \\ -0.2503 \\ 0.2151 \\ 0.5067 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -0.0501 \\ 0 \\ 0 \end{Bmatrix} \tag{36}$$

Solving Eq. (36) using Eqs. (19) and (20) yields a diagonal matrix $[\Delta P]$. The diagonal entries of $[\Delta P]$ are the stiffness parameter perturbations

$$\{\Delta k_1, \Delta k_2, \Delta k_3, \Delta k_4\} = \{0, 0.2, 0, 0\} \tag{37}$$

which are exactly the perturbations between the nominal stiffness parameters of Eq. (30) and the perturbed stiffness parameters of Eq. (33).

Experimental Application

To demonstrate the validity of the MREU procedure, the method is applied to data from the NASA dynamic scale model technology (DSMT) program of Langley Research Center.[20] The structure is an eight-bay truss mounted in a cantilevered configuration, as shown in Fig. 2. A series of modal tests was performed on the structure with various structural members removed to simulate different instances of damage. The data sets from this test have been analyzed by many different researchers (see, for example, Refs. 11 and 15). This data set is used to demonstrate the validity of the MREU procedure because it is known to be well characterized and because a full measurement set is available, allowing the update procedure to be considered separately from any DOF reduction or expansion techniques.

The structure was modeled in ABAQUS,[21] using 109 truss rod elements and 36 nodes (4 restrained), for a total of 96 DOF. The element parameters selected for perturbation are the Young's moduli of the longerons and battens in bays 6, 7, and 8 (where bay 8 is closest to the cantilever) and the diagonals in bay 6, for a total of 26 perturbed parameters. The first damage case studied involved the removal of longeron 46 in bay 8 (denoted damage case a in Ref. 20), and the second damage case involved the removal of two members in bay 6—longeron 35 and diagonal 99 (denoted damage case o in Ref. 20). The relevant elements for these two damage scenarios are shown in Fig. 2.

The frequencies of the identified modes are shown in Table 2. The sequence of the modes for each damage case is determined by correlation of the mode shapes in the damaged case with the mode shapes from the undamaged case. For damage case 46, mode 2 is the only measured mode that undergoes a significant shift in frequency, and so it will be used to compute the parameter perturbations for this damage case. For damage case 35/99, modes 2 and 3 undergo

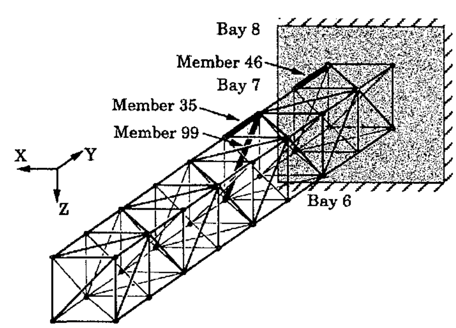

Fig. 2 NASA DSMT eight-bay truss.

Table 2 Comparison of damaged and undamaged modal frequencies for DSMT eight-bay truss

Mode	Undamaged freq., Hz	Damage 46 freq., Hz	Damage 35/99 freq., Hz
1	13.9	13.9	13.7
2	14.5	9.5	9.8
3	48.4	48.5	36.7
4	64.0	64.1	63.3
5	67.5	65.8	58.9

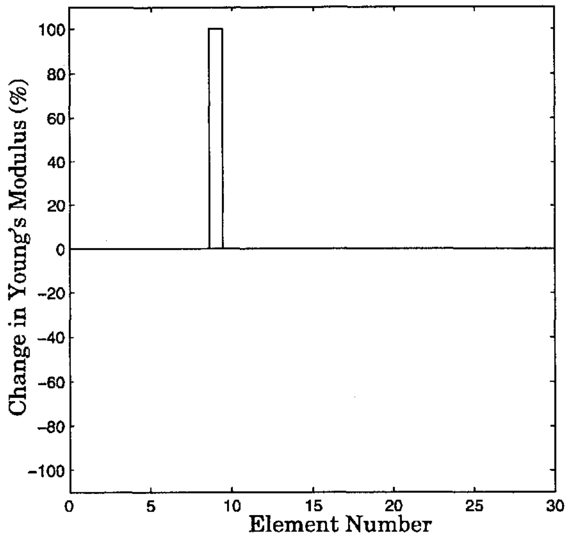

Fig. 3 MREU solution for damage case 46, FEM mode 2.

the largest changes, 32 and 24% respectively. These two modes change much more significantly than the others (the next closest is mode 5 at 13%), and so they will be used to compute the parameter perturbations for that damage case. Observing that modes 2 and 3 are, respectively, a first bending and a first torsion mode, it is hypothesized that the damaged elements are in the half of the truss closest to the cantilever, because this is where such modes will produce the greatest strain energy. This observation is the basis for selecting the 26 parameters that are perturbed.

A minimum-norm procedure was implemented to use as a basis for comparison to the MREU procedure. The minimum-norm algorithm computes the least-squares solution to Eq. (11) to obtain the parameter perturbation vector $\{\Delta p\}$ and thus is similar to the methods presented by Chen and Garba[14] and Li and Smith.[15]

To validate the implementation of the MREU and the minimum-norm element stiffness parameter update procedures, the second FEM mode for damage case 46 was used to compute the elemental Young's moduli perturbations. The results of this update are shown in Fig. 3 for the MREU procedure and in Fig. 4 for the minimum-norm update procedure. In this damage case, element 9 corresponds to longeron 46. Thus, a perfect result would have 100% change in E for element 9 and 0% change in E for all other elements. These results are both nearly perfect, and so the algorithms and the stiffness connectivity matrices can be considered to be correct.

The updates resulting from applying the two algorithms to experimentally measured mode number 2 for damage case 46 are shown in Figs. 5 and 6. The result of the MREU, shown in Fig. 5, has a clear indication of nearly 100% reduction in E for element 9, as well as changes of 30% and less for surrounding members. However, the minimum norm update fails to locate the damaged member. It is interesting to note that for damage case 46, going from FEM modes to measured modes introduces some error into the MREU solution (compare Figs. 3 and 5) but causes the minimum-norm solution to go from nearly perfect to completely ambiguous (compare Figs. 4 and 6).

The application of the MREU and minimum-norm update techniques applied to damage case 35/99 are shown in Figs. 7 and 8,

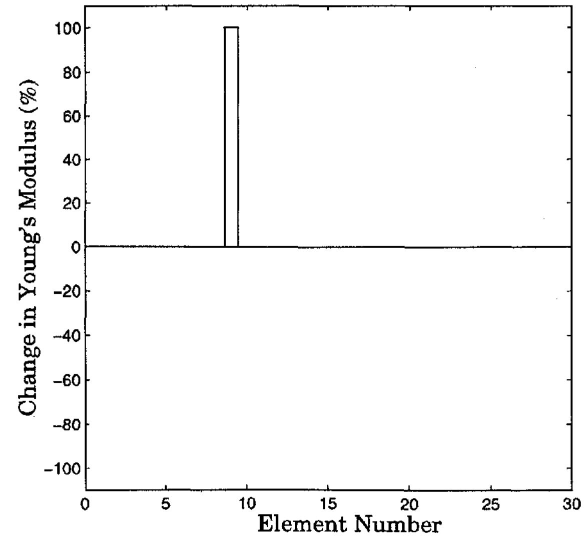

Fig. 4 Minimum-norm solution for damage case 46, FEM mode 2.

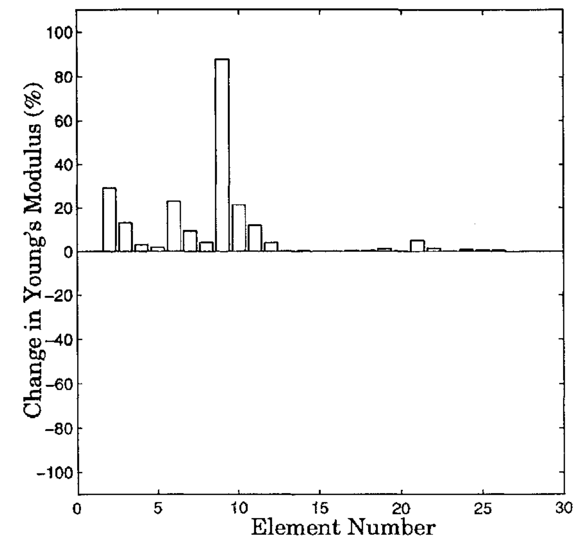

Fig. 5 MREU solution for damage case 46, measured mode 2.

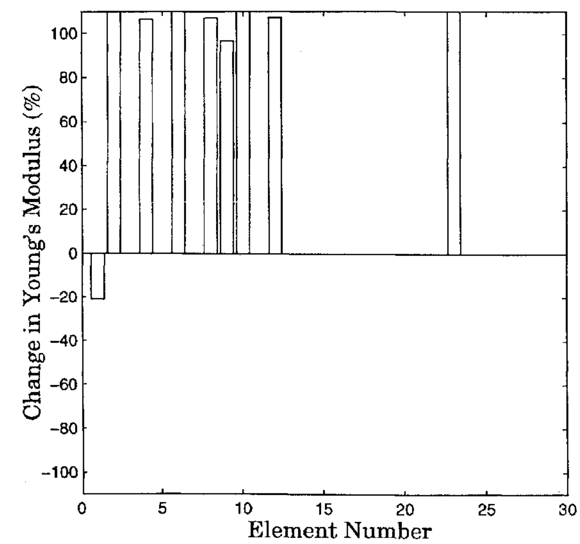

Fig. 6 Minimum-norm solution for damage case 46, measured mode 2.

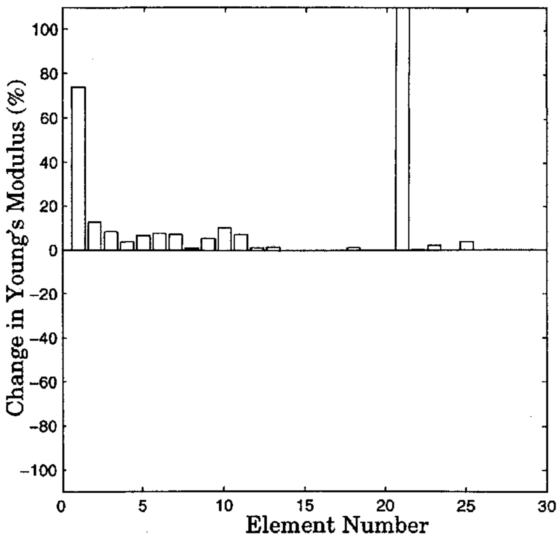

Fig. 7 MREU solution for damage case 35/99, measured modes 2 and 3.

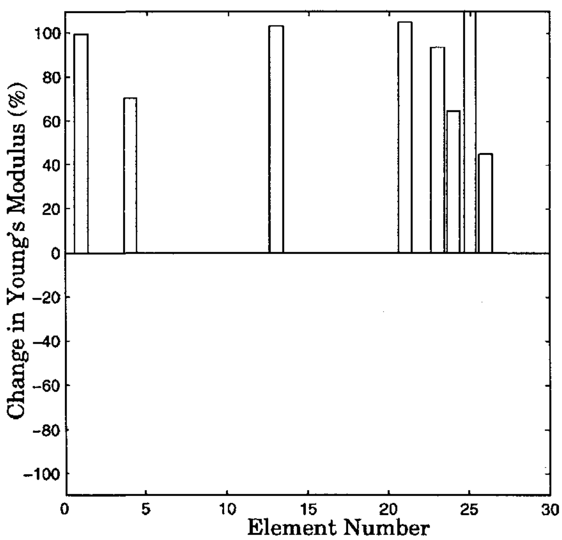

Fig. 8 Minimum-norm solution for damage case 35/99, measured modes 2 and 3.

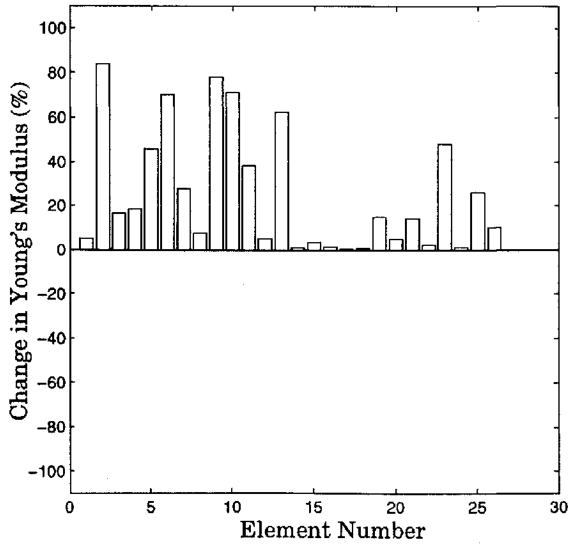

Fig. 9 MREU solution for damage case 46, measured modes 2 and 3.

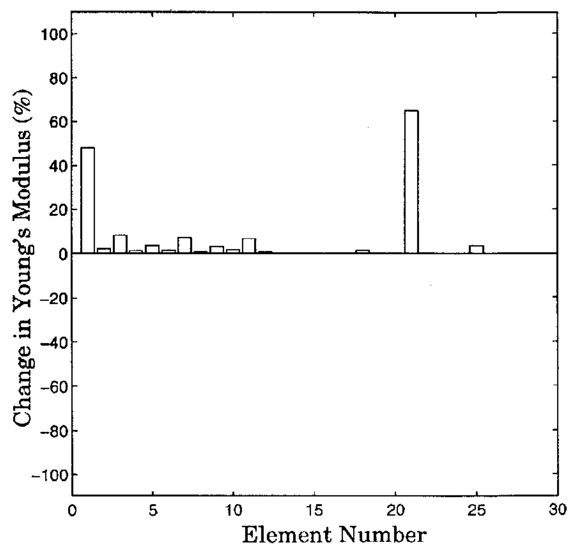

Fig. 10 MREU solution for damage case 35/99, measured mode 2.

respectively. Measured modes 2 and 3 are used for these updates, and the damaged members correspond to elements 1 and 21. As with damage case 46, the results from the MREU technique show two clear peaks at the correct members but also show some smearing at adjacent members. However, the minimum-norm technique again shows large perturbations corresponding to undamaged elements. It is interesting to note that the smearing that occurs in the MREU result consists of perturbations primarily to longerons rather than battens or diagonals. This demonstrates the insensitivity of these particular modes to changes in the stiffnesses of most battens and diagonals.

A known characteristic of any minimum rank update, as reported by Zimmerman et al.,[22] is that the number of modes used in the update is equal to the rank of the computed update property matrix. Thus, using two modes will give an updated property matrix or vector with rank two. However, since the MREU update is equal to the vector that is the diagonal of the solution matrix from the minimum-rank equation, it is not strictly constrained to follow this rule. The results seen from the application of the MREU technique to this experimental data set seem to support the assertion that the best results are obtained when the number of modes equals the number of nonzero perturbations in the parameter vector (which is not necessarily equal to the rank of the perturbation matrix, as stated earlier). For example, consider the damage 46 MREU result using only mode 2 shown in Fig. 5 as compared with the damage 46 MREU result using modes 2 and 3, shown in Fig. 9. The two-mode update of Fig. 9 is completely ambiguous in terms of indicating damage. Likewise, comparison of the damage 35/99 MREU result using modes 2 and 3 in Fig. 7 to the damage 35/99 MREU result using only mode 2 in Fig. 10 shows that the actual stiffness changes in the damaged members (100% change in elements 1 and 21 on the plots) are detected more accurately using two modes with only slightly more smearing. Thus, the optimal number of modes to use in the MREU seems to be equal to the expected rank of the elemental stiffness perturbation vector. This is a potential drawback to any minimum-rank optimal matrix update technique, because in practice the expected order of the damage will typically be unknown.

As a final note on these experimental results, it is widely assumed that sensor DOF 45 in the DSMT eight-bay damage data sets contains erroneous measurements. No attempt to correct this erroneous sensor measurement was made in this analysis.

Conclusions

A new optimal matrix update method was introduced and demonstrated on both numerical and experimental data. The method computes a minimum-rank vector of perturbations to the element-level stiffness parameters while constraining the connectivity of the global stiffness matrix. The motivation for the method was shown in the context of existing optimal matrix update methods. The method was able to locate the damaged members using data from the NASA Langley Research Center eight-bay truss damage detection experiments for both single-member and multiple-member damage cases. The results indicate that the method may be useful for modification of dynamic finite element models in the context of damage detection and model refinement.

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